Complex flows with transport at the pore scale in porous media

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Abstract.
This communication presents a numerical strategy to compute efficiently the Stokes-Brinkman flow for a complex fluid whose rheological properties depend on the concentration of a miscible agent. The numerical algorithm has a quasi-linear computational cost which is very attractive for 3D computations and equivalent permeability computations in porous media.

When a fluid pass a porous medium, the flow at the pore scale can mathematically be described by the Stokes-Brinkman problem. In this work a complex fluid with a generalized Newtonian rheology is considered. The associated rheological parameters can depend on an active constituent of the fluid which is the solution to a transport equation (with eventually diffusion and/or...
reaction terms). Thus the following problem is solved:

\[
\begin{aligned}
- \text{div}(\mu_g D(u)) + \frac{\mu_g u}{K} + \nabla p &= f \quad \text{in } \Omega \\
\mu_g(u, \alpha) &= \kappa(\alpha) \dot{\gamma}(u) \quad \text{in } \Omega \\
\text{div} u &= 0 \quad \text{in } \Omega \\
\frac{\partial \alpha}{\partial t} + u \cdot \nabla \alpha - \eta \Delta \alpha &= s(\alpha) \quad \text{in } \Omega
\end{aligned}
\]

(1)

with adequate boundary conditions. In these equations \( \Omega \) is the computational domain, \( u \) is the velocity field, \( p \) is the pressure, \( D = (\nabla u + \nabla u^T)/2 \) the strain rate tensor, \( K \) is the permeability tensor, \( \alpha \) is the active constituent concentration, \( \mu_g \) is the generalized viscosity (written here for a power law fluid with rheological parameters \( n \) and \( \kappa \) which depend on \( \alpha \), but different constitutive laws can be chosen) and \( \dot{\gamma}(u) = 2\sqrt{D(u) : D(u)} \).

An iterative algorithm for the variable viscosity penalized Stokes problem [1], originally developed for biological applications, is used in this context of porous media flows. It is based on the use of fast solvers on Cartesian grids (whose computational cost is quasi-linear with respect to the number of grid points) and algebraic perturbation techniques [2]. The methodology is adapted to solve (1), since the penalization term is replaced here by the permeability tensor [4]. Particle methods [3] were chosen to compute transport phenomena: they are robust and their computational cost is linear with respect to the number of grid points. Hence the computational algorithm is particularly adapted for large 3D simulations.

These different algorithms were implemented to simulate the flow on several geometries using up to \( 1024^3 \) grid points. Thus very small cracks and inclusions can be considered. Several example will be shown during the presentation.

Moreover, considering a constant viscosity Newtonian fluid, it is possible to compute the equivalent permeability [5] of a medium. This data can then be reused to compute the flow at a macroscale using the Darcy law.

REFERENCES


